2D FORCE MUSCL SCHEME FOR SIMULATING BREAKING SOLITARY WAVE RUNUP

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ABSTRACT

Breaking wave simulation using depth averaged based model, i.e. shallow water equation (SWE) is still a challenge in tsunami modeling. SWE does not have the ability to capture shock and discontinuity. However, certain numerical scheme may enhance SWE for breaking wave simulation. In this study, a 2D First Order Center Scheme with Monotonic Upstream Scheme of Conservation Laws (FORCE MUSCL) scheme is developed. The model ability to capture shock and handling discontinuity was tested by simulating flow over submerged structure. The expected change from sub-critical to critical flow was observed. The model was benchmarked and applied to simulate breaking solitary wave run up. It was found that the predicted run up height and wave profile agrees well with the experimental data from previous study.

1. INTRODUCTION

It is very important to understand tsunami wave since it may cause devastating effect to coastal area. Numerical simulation is considered to be a valuable tool in assessing tsunami. Shallow water equation (SWE) is one of the most commonly used model. Nevertheless, depth averaged model such as SWE can not accurately assess breaking wave. Breaking wave process requires vertical distribution of velocity which depth averaged model is unable to perform. However, the process can be reproduced numerically, i.e. using artificial dissipation (Hansen, 1962). Unfortunately, artificial dissipation requires pre-calibration, thus it is not favorable.

The use of artificial dissipation is not necessary when employing certain numerical scheme, developed for shock capturing (Li and Raichlen, 2002). Furthermore, finite volume FORCE MUSCL scheme may provide a more robust method for simulating breaking solitary wave run up (Mahdavi and Talebbeydokhti, 2009). Moreover, the method has been further enhanced by employing Simultaneous Coupling Method (SCM), assessing bed stress from boundary layer (Adityawan and Tanaka, 2011). Nevertheless, the method was for 1D computation. Hence, the use of the model is still limited. In this study, 2D FORCE MUSCL scheme is developed and used to simulate flow over submerge structure and breaking solitary wave runup.

2. MODEL DEVELOPMENT

Governing Equations

SWE consists of continuity equation and momentum equation. The continuity equation is as follow.

$$\frac{\partial h}{\partial t}+\frac{\partial hU}{\partial x}+\frac{\partial hV}{\partial y}=0 \tag{1}$$

where h is the water depth, U and V are the depth averaged velocity at the corresponding axis of x and y, and t is time. The momentum equations are as follows.

$$\frac{\partial hU}{\partial t}+\frac{\partial hU^2}{\partial x}+\frac{\partial \frac{1}{2}gh^2}{\partial x}+\frac{\partial hUV}{\partial y}=gh(S_{0x}-S_{fx}) \tag{2}$$

$$\frac{\partial hV}{\partial t}+\frac{\partial hUV}{\partial x}+\frac{\partial hV^2}{\partial y}+\frac{\partial \frac{1}{2}gh^2}{\partial y}=gh(S_{0y}-S_{fy}) \tag{3}$$

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where g is gravity. S_o and S_f are bed slope and friction slope at x and y direction, respectively. Here, friction slope is calculated based on Manning equation as follows.

$$S_{fx} = \frac{n^2 U \,|\sqrt{U^2 + V^2}\,|}{h^{4/3}} \tag{4}$$

$$S_{fx} = \frac{n^2 V \,|\sqrt{U^2 + V^2}\,|}{h^{4/3}} \tag{5}$$

where n is the Manning roughness coefficient.

Numerical Methods

The governing equation can be rewritten in their vector form as follow.

$$\frac{\partial E}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = S \tag{6}$$

with E donates the conservative variables (h, hU, and hV), F donates flux variables in x axis (hU^2+1/2gh^2, hUV), G donates flux variables in y axis (hV^2+1/2gh^2, hUV) and S is the source term. ($gh(S_{ox}$-$S_{fx})$, $gh(S_{oy}$-$S_{fy})$). Thus the solution can be approached as.

$$\frac{\partial V}{\partial t} = \underbrace{-F'(x) - G'(y) + S}_{L} \tag{7}$$

The space derivation of F and G in its corresponding direction can be solved using finite volume center scheme as follows.

$$F'(x_{i,j}) = \frac{F_{i+(1/2),j} - F_{i-(1/2),j}}{\Delta x} \tag{8}$$

$$G'(y_j) = \frac{G_{i,j+(1/2)} - G_{i,j-(1/2)}}{\Delta y} \tag{9}$$

The FORCE MUSCL method with superbee slope limiter (Toro, 2011) is utilize for grid reconstruction and approaching the flux variable value at each axis.

MUSCL

The Monotonic Upstream Scheme of Conservation Laws (MUSCL) method reconstructs the grid system by approaching the conservatives variable value at the left and right side of the cell interface. The following examples are given for MUSCL method to reconstruct the grid for conservative variables of hU and h along x axis. The value of hU at the right (+) and left (-) of the cell interface ($i+1/2$) are approximated as follows.

$$(hU)^{-}_{i+1/2,j} = (hU)_{i,j} + \frac{1}{2}\delta_{i,j}(hU) \tag{10}$$

$$(hU)^{+}_{i+1/2,j} = (hU)_{i+1,j} - \frac{1}{2}\delta_{i+1,j}(hU) \tag{11}$$

Here, the value of $\delta_{i,j}$ at the corresponding node is given by.

$$\delta_{i,j}(hU) = \Psi_{i,j}\Delta_{i,j}(hU) \tag{12}$$

where the value of $\Delta_{i,j}$ is calculated as follow.

$$\Delta_{i,j}(hU) = \frac{(hU)_{i+1,j} - (hU)_{i-1,j}}{2} \quad (13)$$

The slope limiter $\Psi_{i,j}$ in Eq. (12), which is very important in the shock handling computation, is determined based on the value of r, which is calculated as follow.

$$r_{i,j} = \frac{\Delta_{i-1/2,j}(hU)}{\Delta_{i+1/2,j}(hU)} \quad (14)$$

in which,

$$\Delta_{i+1/2,j}(hU) = (hU)_{i+1,j} - (hU)_{i,j} \quad (15)$$

$$\Delta_{i-1/2,j}(hU) = (hU)_{i,j} - (hU)_{i-1,j} \quad (16)$$

The slope limiter value is given based on the value of $r_{i,j}$ as follows.

$$\Psi_{i,j} = \begin{cases} 0 & r_{i,j} \le 0 \\ 2r_{i,j} & 0 \le r_{i,j} \le 1/2 \\ 1 & 1/2 \le r_{i,j} \le 1 \\ \min(r_{i,j}, \frac{2}{1+r_{i,j}}, 2) & else \end{cases} \quad (17)$$

The same method applies to the other conservative variables in both x and y direction, accordingly. However, it should be noted here that surface gradient method is required for the water depth. Here, the water depth variable is converted to the surface elevation, to accommodate bed level change. Surface elevation is calculated as follow.

$$\eta = h + z_b \quad (18)$$

where η is the surface elevation, and z_b is the bed elevation.

FORCE

First Order Center Scheme (FORCE) method is applied to obtain the value of flux F and G at the cell interface based on the estimated value from MUSCL. The FORCE method is basically a combination of Lax-Freidreich and Lax-Wendroff scheme. The following example shows flux F calculation in x direction.

$$F_{i+1/2,j} = \frac{1}{2}\left(F^{LF}_{i+1/2,j} + F^{LW}_{i+1/2,j}\right) \quad (19)$$

$$F^{LF}_{i+(1/2),j} = \frac{1}{2}\left(F^{-}_{i+(1/2),j} + F^{+}_{i+(1/2),j}\right) - \frac{1}{2}\frac{\Delta x}{\Delta t}\left(V^{+}_{i+(1/2),j} - V^{-}_{i+(1/2),j}\right) \quad (20)$$

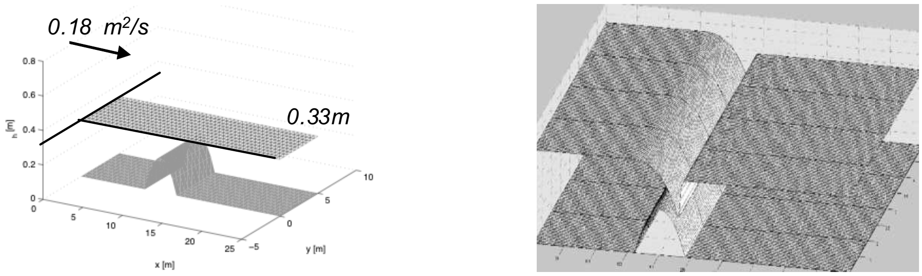
$$F^{LW}_{i+(1/2),j} = F(V^{LW}_{i+(1/2),j}) \quad (21)$$

$$V^{LW}_{i+(1/2),j} = \frac{1}{2}\left(V^{-}_{i+(1/2),j} + V^{+}_{i+(1/2),j}\right) - \frac{1}{2}\frac{\Delta t}{\Delta x}\left(F(V^{+}_{i+(1/2),j}) - F(V^{-}_{i+(1/2),j})\right) \quad (22)$$

Eq. (19) to Eq. (22) are repeated in y direction to obtain the value of flux G.

RUNGE KUTTA 3rd ORDER

Eq.(8) and Eq. (9) can be solved based on the obtained flux value from FORCE method, described previously. Thus, the value of the right hand side (*L*) in Eq. (7) can be obtained. Time derivative to obtain conservative variables at the next time step is solved using Runge Kutta 3rd order scheme with auto time step ensuring that the Courrant number value does not exceed 0.8, as follows.



(a) Model setup (b) Simulation result at T=20s

Fig. 1 Flow over submerge structure

$$V_{i,j}^{t+1} = 3/4 \times V_{i,j}^{t} + 2/3 \times V_{i,j}^{(2)} + 2/3 \times \Delta t \times L(V_{i,j}^{(2)}) \tag{23}$$

where,

$$V_{i,j}^{(1)} = V_{i,j}^{t} + \Delta t \times L(V_{i,j}^{n}) \tag{24}$$

$$V_{i,j}^{(2)} = 3/4 \times V_{i,j}^{t} + 1/4 \times V_{i,j}^{(1)} + 1/4 \times \Delta t \times L(V_{i,j}^{(1)}) \tag{25}$$

Moving Boundary Condition

Wave runup simulation requires a moving boundary condition to separate the wet cell and the dry cell in the computation. The moving boundary condition in this method is given by stating a minimum threshold for the water depth in all dry cell. The water depth is equal to this threshold value with zero velocities (both direction) if the calculated water depth is lower than the threshold value.

3. RESULTS AND DISCUSSIONS

The developed model ability to capture shock and discontinuity was verified by simulating flow over submerged structure (Ern et al., 2000). A structure is located in the bed of a flume. The submerged structure follows the following equation.

$$z_b(x, y) = \max\left(0, 0.02 - 0.05\left(x - 10\right)^2\right) \tag{26}$$

The initial water elevation at the flume is given at 0.33 m with the minimum bed at 0, with zero velocity. Incoming flow with the velocity of 0.18 m^2/s is given at one side of the flume. The model setup is shown in Fig. 1 (a).

Simulation was conducted for 20 seconds. The result is shown in Fig. 1 (b). The final results shows that the model is able to simulate shock in case of flow over submerged structure, resulting from supercritical to critical condition. Good comparison are found to the previous study. The flow is relatively stable after 20 seconds, with the upstream final height of 0.42 m, downstream jump location at 11m and the minimum depth near the structure is 0.17 m.

The developed model was further employed to simulate solitary breaking wave run up (Synolakis, 1986) on a sloping beach (1:20). The model setup is shown in Fig. 2. with H is the incoming wave height and h_o is the initial water depth. The case has been widely used and accepted as model benchmark. The breaking wave case is given for the ratio of H/h_o = 0.3. The incoming wave height follows the following equation.

$$\eta(x,0) = H \times \text{sech}^2\left(\sqrt{\frac{3H}{4h_o^{\ 3}}}(x - x_1)\right) \qquad (27)$$

All variables for comparison are given in non-dimensional. Detail explanations can be found in the reference (Synolakis, 1986).

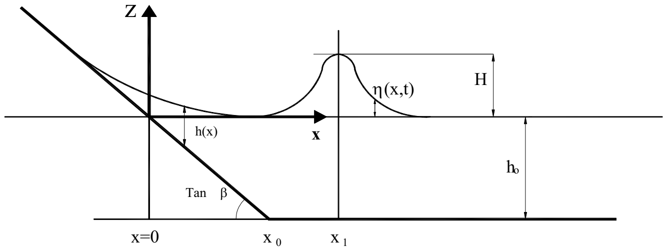
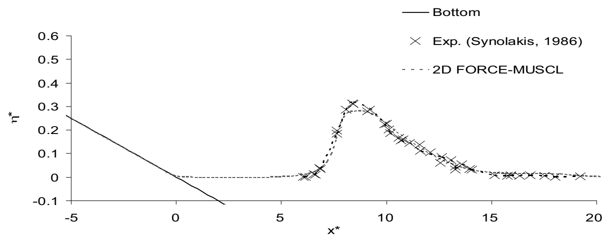
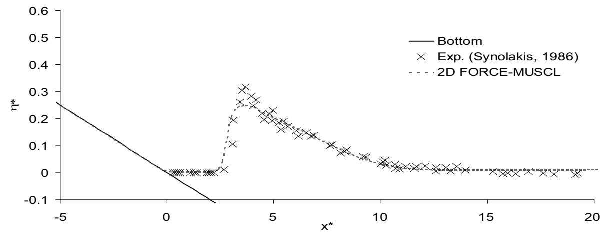


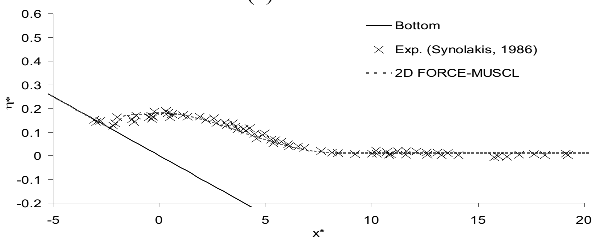
Fig. 2 Breaking wave runup model setup



(a) t* = 15



(b) t* = 20



(c) t* = 25

Fig. 3 Surface profile (centerline) (H/h_o=0.3)

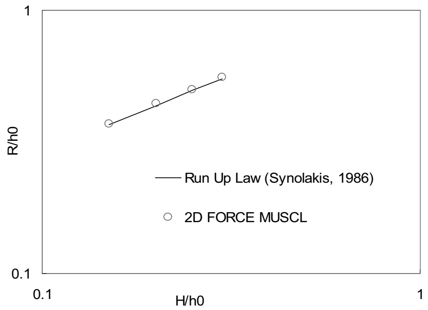


Fig. 4 Runup height

The water level profile at the centerline is shown in Fig. 3. Furthermore, several other cases were simulated by changing the H/h_o ratio. The predicted runup height from these cases are shown in Fig. 4. Both, surface profile and predicted runup height show good comparison with the previous study (Synolakis, 1986).

4. CONCLUSIONS

2D FORCE MUSCL has been successfully developed and used to simulate flow over submerge structure and breaking solitary wave runup. The shock capturing ability of the model was verified. The model is able to handle shock due to change from subcritical to supercritical flow condition in the case of flow over submerged structure. Moreover, the model successfully simulates the breaking solitary wave runup benchmark case with good comparison in both, surface profile and predicted runup height.

Future development of the model will require further verification with non-uniform bed in both axis or other cases with multidirectional flow. In addition, the model can be further enhanced by using SCM, for a more accurate bed stress assessment from the boundary layer.

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