Analysis on morphological recovery of tsunami-induced concave shoreline

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Abstract
The 2011 tsunami caused concave shoreline around the Nanakita River mouth area, Sendai Coast, Miyagi Prefecture, Japan. The recovery process of morphology in this area is much depended on the longshore sediment which is transported from adjacent sandy coasts. Coastal structures on both sides of the concave portion are considered as rigid boundaries, and have significant influence on the evolution of shoreline. Analytical solutions of one-line model, which describe the evolution of shoreline around the concave portion in cases without and with rigid boundaries, have been discussed. The obstruction to the shoreline evolution is revealed through the analysis of analytical solutions. Results show that dimensionless recovery time from two solutions is asymptotic when the ratio of concave width to the total length of sandy coasts bounded by two rigid boundaries is small. Analytical solution of one-line model for estimating the area of sand deposition in the concave portion is introduced.

1. Introduction

Concave shoreline at river mouth areas is the common morphology observed right after the 2011 tsunami. It was formed after the flushing of river mouth sand spit and the eroding of sand barrier adjacent to the river mouth by the tsunami (Tanaka et al., 2012). The recovery process of this kind of morphology has been presented by Hoang et al. (2015). In that study, the evolution of shoreline positions after the tsunami at the Nanakita River mouth and the Akaiko Breaching areas has been reported. Analytical solution of one-line model, which describes the evolution of shoreline in case without rigid boundaries on both sides of the concave portion, is given by Larson et al. (1987). Moreover, an analytical solution of one-line model describing the evolution of concave shoreline on the coast bounded by rigid boundaries has been introduced by Hoang et al. (2015). Dimensionless recovery time of morphology, which is depended on the ratio of total length of adjacent sandy coast to the concave width, has been presented. However, the correlation between these solutions hasn't been discussed yet. Moreover, the obstruction of rigid boundaries such as jetty, breakwater on the evolution of shoreline hasn't been discussed clearly.

During the recovery process, longshore sediment is transported into the concave portion. This sediment is supplied from the adjacent sandy coasts. Volume of sediment transported into the concave portion in corresponding to elapsed time is necessary to evaluate. This theory is important not only for coastal management but also for the engineering. It can be applied in the field of beach nourishment. Dean (2003) mentions that the evolution of shoreline position on the beach nourishment in case of pocket beach is not important, because the final equilibrium state is known in advanced. However, knowing the volume of sediment remained after a certain time of nourishment is important and useful.

Hence, this study would like to discuss

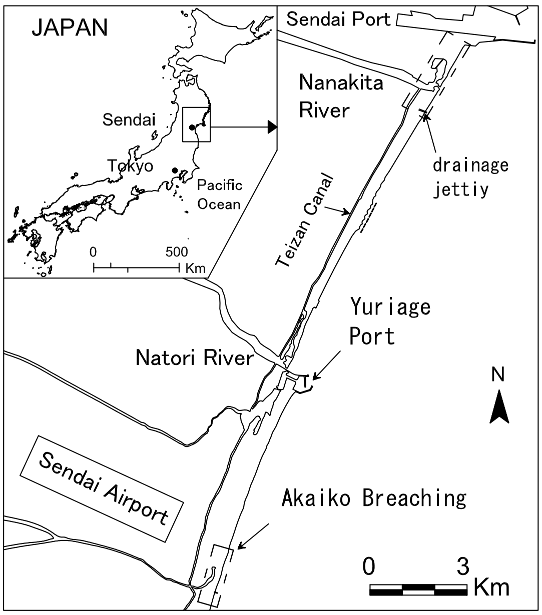

Fig. 1 Location map of study area

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further on analytical solutions for the recovery process of concave shoreline formed after the tsunami, and estimate the volume of sediment deposition in the concave portion through the analytical solution of one-line model.

2. Study area and data collection

This study focuses on area around the Nanakita River mouth which is located on the northern part of Sendai Coast, Miyagi Prefecture, Japan (Fig. 1). There are a long breakwater of Sendai Port at north and drainage of waste water treatment plant at south. These structures are considered as rigid boundaries.

In addition, the Akaiko Breaching, which is located approximately 16.5km south of the Nanakita River mouth, is also taken as the study area. This breaching was formed by the tsunami at the location of an old river mouth. Although this area was not river mouth strictly, this breaching was temporarily playing the geographical features as a river mouth after the tsunami.

Aerial photographs of study area have been taken frequently in every one or two month since 1990. All raw aerial photographs are georeferenced to the World Geodetic System (WGS-84). In addition, aerial photographs of the Akaiko Breaching were collected from Geospatial Information Authority of Japan (GSI) and Google Earth.

3. Results and discussion

(1) Morphological recovery after the tsunami

Aerial photographs in Fig. 2 show the recovery of morphology at the Nanakita River mouth area. Before the tsunami, shoreline on both sides of the Nanakita River mouth is in dynamic equilibrium (Pradjoko and Tanaka, 2010). Morphology in this area was severely damaged by the tsunami. The concave shoreline can be observed after the tsunami [Fig. 2(b)]. The width of concave poriton is about 1km. There is sufficient sediment supply from adjacent sandy coasts, the recovery of this concave area was quite fast. The dimensionless recovery time has been presented depending on the ratio of total length of adjacent sandy coasts to the concave width (Hoang et al., 2015). The closure of the Nanakita River mouth was observed in September 2011 [Fig. 2(d)]. Aerial photographs in Fig. 3 show the

(a) Mar 6, 2011

(b) Mar 14, 2011

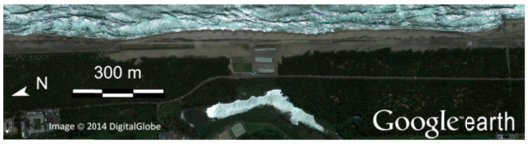
(c) Apr 6, 2011

Fig. 2 Morphological recovery at the Nanakita River mouth after the tsunami

(d) Sep 7, 2011

(e) Apr 8, 2012

Fig. 2 Continued



(a) Aug 14, 2009

(b) Mar 14, 2011

(c) Apr 6, 2011

(d) May 3, 2011

(e) Apr 10, 2012

Fig. 3 Morphological recovery at the Akaiko Breaching after the tsunami

recovery process of morphology at the Akaiko Breaching. The breaching was induced by the tsunami at the old river mouth location. The width of concave portion is more than 100m. The recovery time in this area is shorter than at the Nanakita River mouth area.

(2) Analysis on analytical solution describing the evolution of concave shoreline

During the recovery of morphology, sediment on adjacent sandy coasts is transported to the concave portion. In this case the longshore sediment is predominant, thus, the evolution of shoreline can be described by one-line model. Governing equation of one-line model can be simplified as diffusion equation, Eq. (1), based on following simplifications. The angle of wave breaking crests to local shoreline is supposed to be small, and breaking wave height is assumed to be constant along the coast.

$$\frac{\partial y}{\partial t} = \varepsilon \frac{\partial^2 y}{\partial x^2} \tag{1}$$

where x and y are the coordinates which are defined as in Fig. 4; t is the time; ε is the diffusion coefficient.

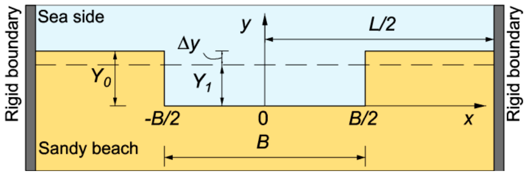


Fig. 4 Schematic diagram of bounded rectangular beach cut

The evolution of shoreline position around the concave portion can be described by analytical solution of one-line model. Larson et al. (1987) and Hoang et al. (2015) introduce solutions for cases without and with rigid boundaries at both ends as Eqs. (2) and (3), respectively.

$$y = \frac{1}{2}Y_0\left[\mathrm{erfc}\left(\frac{B\text{-}2x}{4\sqrt{\varepsilon t}}\right)+\mathrm{erfc}\left(\frac{B+2x}{4\sqrt{\varepsilon t}}\right)\right] \quad (2)$$

$$y = Y_0\left[1\text{-}\frac{B}{L}\text{-}\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi B}{L}\exp\left(\text{-}\frac{4\varepsilon n^2\pi^2 t}{L^2}\right)\cos\frac{2n\pi x}{L}\right] \quad (3)$$

where Y_0 is the cross-shore distance of beach cut region from the initial shoreline. This distance is estimated based on the actual condition of shoreline right after the tsunami; *erfc* is the complementary error function; B is the width of concave portion; L is the total length of sandy coast bounded by two rigid boundaries.

Eqs. (2) and (3) are expressed in dimensionless form as Eqs. (4) and (5), respectively.

$$y^{*}= \frac{1}{2}\left[\mathrm{erfc}\left(\frac{1\text{-}2x^{*}}{4\sqrt{t^{*}}}\right)+\mathrm{erfc}\left(\frac{1+2x^{*}}{4\sqrt{t^{*}}}\right)\right] \quad (4)$$

$$y^{*}=1\text{-}B^{*}\text{-}\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\pi B^{*}\exp\left(\text{-}4n^2\pi^2 t^{*}B^{*2}\right)\cos 2n\pi x^{*}B^{*} \quad (5)$$

where the dimensionless parameters are defined as follows.

$$y^{*}=\frac{y}{Y_0} \quad (6)$$

$$x^{*}=\frac{x}{B} \quad (7)$$

$$t^{*}=\frac{\varepsilon t}{B^2} \quad (8)$$

$$B^{*}=\frac{B}{L} \quad (9)$$

The recovery time, T_E, is defined to be the time when shoreline position at central line (x=0m) becomes 99% of the equilibrium shoreline position. The equilibrium shoreline positions of cases without and with rigid boundaries are Y_0 and Y_1, respectively. Value of Y_1 is obtained from the Eq. (10).

$$Y_1=\frac{Y_0(L\text{-}B)}{L} \quad (10)$$

By utilizing Eqs. (4) and (5) and above conditions, relationship between the dimensionless concave width

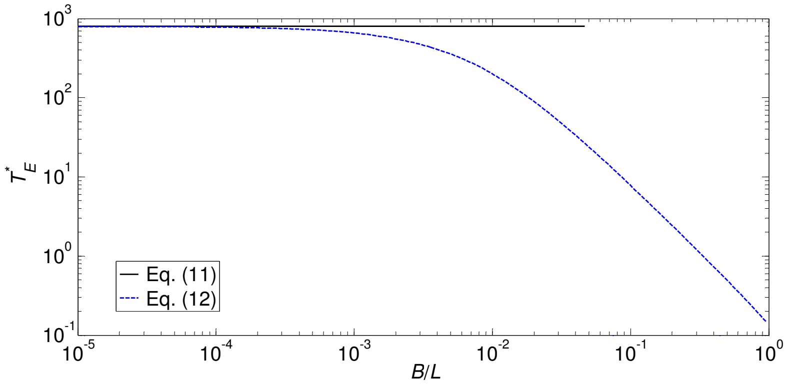


Fig. 5 Relationship between T_E* and B/L

and the dimensionless recovery time of two solutions is given by the following equations.

$$0.99=\mathrm{erfc}\left(\frac{1}{4\sqrt{T_E^*}}\right) \tag{11}$$

$$0.99\left(1-B^*\right)=1-B^*-\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\pi B^*\exp\left(-4n^2\pi^2 T_E^* B^{*2}\right) \tag{12}$$

where the dimensionless recovery time is defined as follows.

$$T_E^*=\frac{\varepsilon T_E}{B^2} \tag{13}$$

In this study, the dimensionless parameters x*, t*, B* and T_E* are defined in different way compared to Hoang et al. (2015).

Figure 5 shows the relationship between the ratio of concave width to the length of the coast bounded by rigid boundaries (B/L) with the dimensionless recovery time. The dimensionless recovery time, which is obtained from the solution for the case without rigid boundaries, is always constant, whereas, the dimensionless recovery time is getting smaller when B/L value is getting larger in the case of solution with rigid boundaries. However, the recovery time obtained from these solutions is asymptotic when the ratio of B/L is rather small. It is common to observe in the beach nourishment that fixed boundaries are constructed on both sides of the nourishment area to prevent the leaving out of sediment. Thus, results in Fig. 5 are also valid for this case.

(3) Sediment movement into the concave portion

The volume of sediment, which is transported into the concave portion, can be obtained by multiplication the area of sand deposition and the depth of closure. The area of sand deposition, A_D, is the area on the concave portion plane where sediment has been deposited. Equations for estimating the area of sand deposition for the case with rigid boundaries is given as the following equation.

$$A_D=2Y_0\frac{L}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin^2\frac{n\pi B}{L}\left[1-\exp\left(-\frac{4\varepsilon n^2\pi^2 t}{L^2}\right)\right] \tag{14}$$

The concave shoreline at the Nanakita River mouth (Case 1) and the Akaiko Breaching (Case 2) areas are typical cases of concave shorelines after the tsunami. Eq. (14) is utilized for these cases. The values of B, Y_0, L and ε for these cases are 1040m, 68m, 2350m, 320m^2/day and 120m, 270m, 1600m, 340m^2/day, respectively.

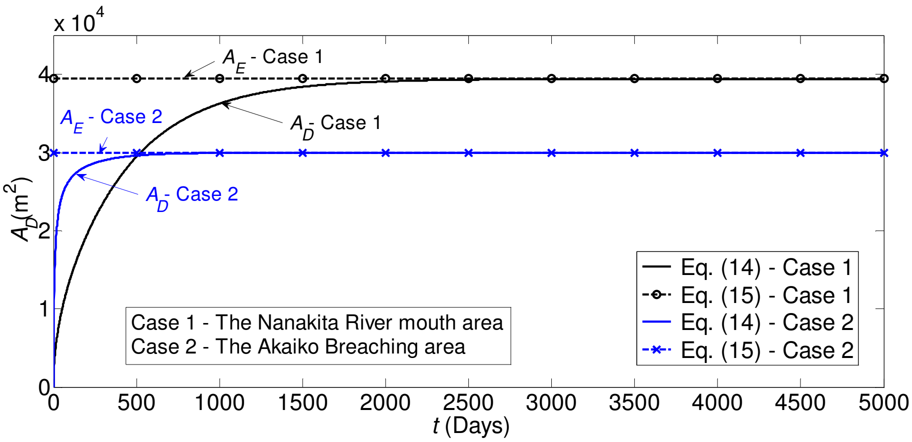


Fig. 6 Evolution of the area of sand deposition in the concave portion at the Nanakita River mouth and the Akaiko Breaching areas

On the other hand, the area of sand deposition when shoreline position getting equilibrium state for the case with rigid boundaries is given as the following equation.

$$A_E = \frac{Y_0 (L\text{-}B)}{L} B \tag{15}$$

Figure 6 shows the simulated results of the area of sand deposition corresponding to elapsed time for Case 1 and Case 2. In the early period, the area of sand deposition is increasing. When increasing time, it approaches the equilibrium area of sand deposition (A_E). The area of sand deposition of Case 1 is larger than Case 2.

It is interested to note that, by changing the side of Y_0, Eq. (14) can estimate the area (or volume) of sand remaining after a certain time of beach nourishment.

4. Conclusions

This study has discussed on the analytical solutions of one-line model which describe the evolution of shoreline around the concave portion in case without and with rigid boundaries. The dimensionless recovery times obtained from analytical solutions of one-line model for the cases without and with rigid boundaries are asymptotic when the ratio of B/L is small. An equation for estimating the area (or volume) of sand deposition in the concave portion is introduced. This is very important not only for researching but also for engineering. Simulated results of the area of sand deposition corresponding to elapsed time on cases of the Nanakita River mouth and the Akaiko Breaching areas are presented.

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