River delta formation in response to the effect of boundaries located at a distance from the river mouth

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Abstract: A comparison has been made between two analytical solutions of Larson et al. (1987) and Tanaka et al. (2017) to observe the effect of the boundary to the formation process of a river delta. Measured shoreline positions at Funatsu River delta in Lake Inawashiro between 1982 and 2015 were also used to validate the analytical solution of Tanaka et al. (2017).

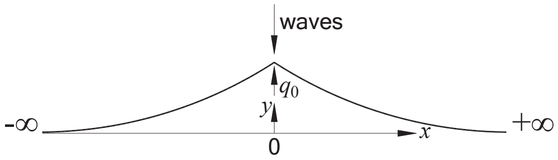

Figure 1. Schematic diagram of infinite delta shorelines (Larson et al., 1987).

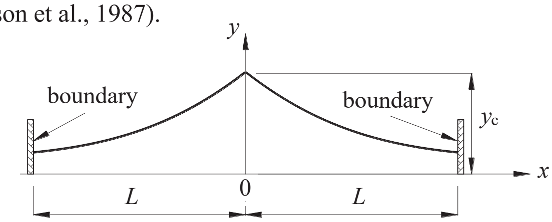

Figure 2. Schematic diagram of infinite delta shorelines (Tanaka et al., 2017).

1. Introduction

The river deltas have been recognized as natural resources for the activities of human beings (Refaat, 1990). However, substantial changes of river delta coastlines have been reported globally in the recent years (Uda, 2010; Viet et al., 2015; Ali and Elmagd, 2016; Fan et al., 2018). Therefore, studying the evolution of delta coastlines is crucial since it provides essential information for understanding the coastal response to many complex processes (Jones et al., 2009).

Coastal scientists and engineers have long sought a robust and practical methodology for the prediction of shoreline change along sandy beaches (Davidson et al., 2013). In which, the conservation-of-sand-volume approach, also known as the one-line approach, has remained the preferred model for simulating long-term shoreline evolution (Thomas and Frey, 2013). Since the first mathematical model of Pelnard Considere (1956), numerous models have been developed with various approaches and approximations.

In order to rapidly and economically estimate the formation process of the river delta shorelines owing to river-borne sediment (q_0), Larson et al. (1987) introduced an analytical solution derived from a simplified equation of one-line model. This analytical solution is applicable for infinite river delta shorelines (Figure 1). In which, x is the alongshore distance and y is the offshore distance. In reality, however, a delta shoreline is always limited in an extent between the river mouth and a boundary (e.g., coastal structures, headlands). Therefore, Tanaka et al. (2017) provided another analytical solution which

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is useful for studying the formation processes of river deltas with finite shorelines as shown in Figure 2. In which, y_C is the maximum shoreline position determined as a distance between the x-axis and the delta's tip, L is the length of the delta shoreline.

Although experimental data was used to validate the analytical solution of Tanaka et al. (2017), there is no application of this solution to a specific study area. In order test the applicability of the solution provided by Tanaka et al. (2017), this study will utilize measured data at Funatsu River delta in Lake Inawashiro. Before validating, a theoretical discussion will be made to observe the characteristic change of delta shorelines in response to the effect of a boundary located at the distance L from the river mouth.

2. Study area and data collection

This study will take Funatsu River delta in Lake Inawashiro as a case study. An outline of Lake Inawashiro is shown in Figure 3. As can be seen from Figure 3, shoreline on the left of Funatsu River mouth is finite due to the existence of a pier. Therefore, this shoreline is suitable for applying the solution of Tanaka et al. (2017). In addition, a photo taken in the field trip on Nov 10, 2017 clearly shows the effect of the boundary to shoreline orientation. In which, the shoreline is almost a straight line near the boundary.

A series of aerial and satellite images from 1982 to 2015 in Funatsu River delta will be used for the analysis. All the images are rectified to a same coordinate system.

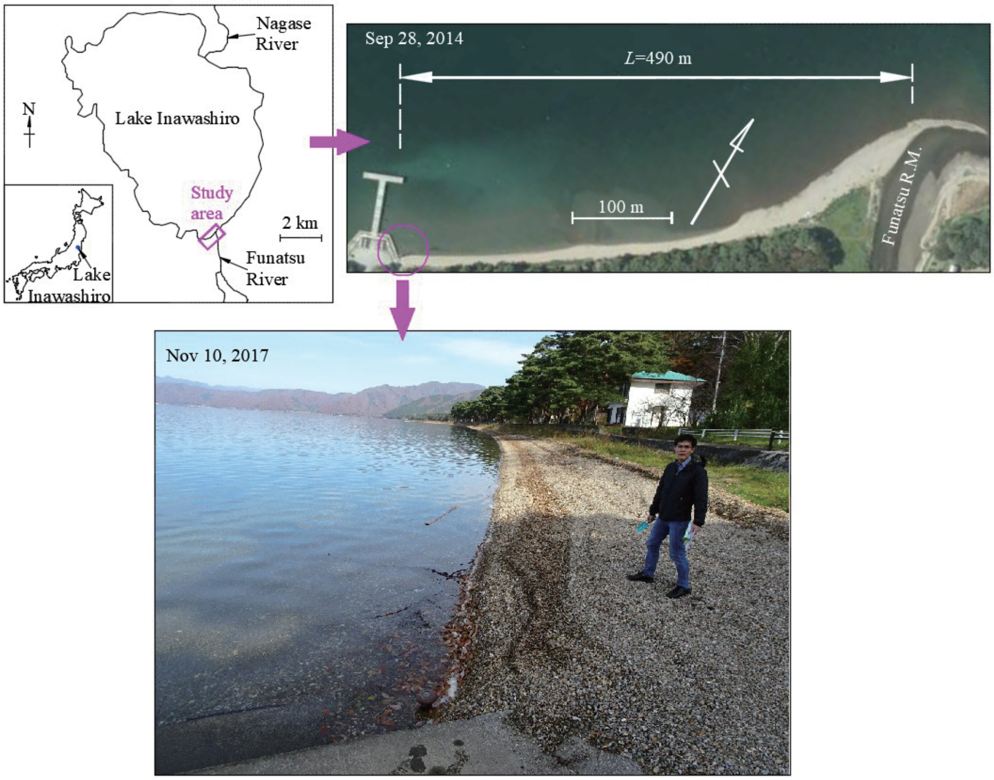

Figure 3. The outline of Lake Inawashiro and effect of the boundary on the left shoreline at Funatsu River mouth.

3. Results and discussion

3.1. Analytical solution for the formation processes of finite river delta shorelines

The development process of a river delta bounded by two structures (Figure 2) was discussed using the approach of the one-line model with the simplified governing equation (Larson et al., 1987):

$$\frac{\partial y}{\partial t} = \varepsilon \frac{\partial^2 y}{\partial x^2} \tag{1}$$

Here t is the time, ε is the diffusion coefficient.

With reference to the solution for heat conduction provided by Myers (1971), Tanaka et al. (2017), derived a new analytical solution of finite shoreline change based on Eq. (1) as:

$$y^* = \frac{x^{*2}}{2} - |x^*| + \frac{1}{3} + t^* - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{e^{-n^2 \pi^2 t^*}}{n^2} \cos\left(n \pi x^*\right) \tag{2}$$

In which, the dimensionless representations of the shoreline position y, alongshore distance x, and time t are as follows.

$$y^* = y \frac{2 \varepsilon D}{q_0 L} \tag{3}$$

$$x^* = \frac{x}{L} \tag{4}$$

$$t^* = \frac{\varepsilon t}{L^2} \tag{5}$$

In order to make a comparison, the analytical solution provided by Larson et al. (1987) is also transformed into the dimensionless form using the dimensionless quantities in Eqs. (3), (4), and (5).

$$y^* = 2\sqrt{\frac{t^*}{\pi}} e^{-\left(x^{*2}/4t^*\right)} - |x^*| \operatorname{erfc}\left(\frac{|x^*|}{2\sqrt{t^*}}\right) \tag{6}$$

Here, erfc is the complementary error function.

Figure 4 shows the shoreline positions plotted using Eqs. (2) and (6). Since the solution is symmetric with respect to the y-axis, the solution for only one side of the symmetry line is displayed. As can be seen from the figure, when the dimensionless time t^* is smaller than 0.1, Eqs. (2) and (6) are perfectly consistent. Around $t^*=0.2$, a difference starts to appear at the right end boundary. However, there is no difference at the river mouth. Thereafter, the difference between the two solutions has expanded, and the influence of the boundary can be observed clearly. After $t^*=0.4$, the shoreline of parabolic shape is moving forward in the offshore direction.

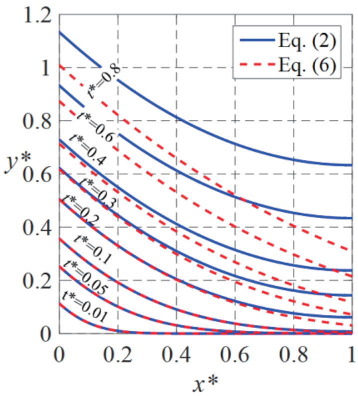

Figure 4. Shoreline evolutions with and without effect of the boundary

In order to investigate the effect of the boundary with the pass of time, shoreline evolutions at the river mouth and the boundary will be plotted.

By substituting $x^*=0$ into Eq. (2), the shoreline position at the river mouth y_0^* can be expressed as a function solely of time t^*

$$y_0^* = t^* + \frac{1}{3} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2 t^*}}{n^2} \tag{7}$$

It is already observed in Figure 4 that when t^* is small, there is no effect of the boundary and the shoreline positions can be described using the analytical solution as in Eq. (6). Therefore, when t^* is small, the shoreline evolution at the river mouth ($x^*=0$) is represented as:

$$y_0^* = 2\sqrt{\frac{t^*}{\pi}} \tag{8}$$

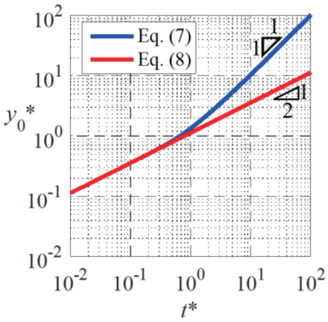

Figure 5. Shoreline evolution at the river mouth

Shoreline evolutions at the river mouth expressed in Eqs. (7) and (8) are plotted in Figure 5. As can be seen in this figure, the transition time indicating the effect of the boundary occurs near $t^*=0.3$. Since then, the effect becomes clear which indicated by the slopes of the blue and red lines.

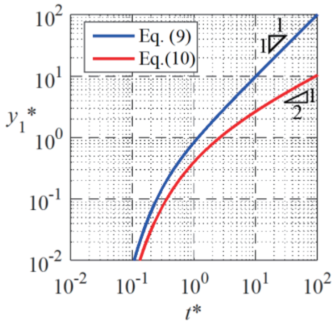

Figure 6. Shoreline evolution at the boundary

Continuing with the comparison, shoreline evolution at the boundary will be investigated. At the boundary, $x=L$ or $x^*=1$, Eq. (2) becomes:

$$y_1^* = t^* - \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}(-1)^n\frac{e^n}{n^2} \tag{9}$$

And Eq. (6) becomes:

$$y_1^* = 2\sqrt{\frac{t^*}{\pi}}e^{-\left(\frac{1}{4t^*}\right)} - erfc\left(\frac{1}{2\sqrt{t^*}}\right) \tag{10}$$

Figure 6 shows the shoreline evolutions at the boundary presented by Eqs. (9) and (10). The difference in these two equations can be seen around $t^*=0.1$. Taking the boundary effect into consideration, this boundary blocks the sediment and causes the shoreline to advance more rapidly (blue line).

3.2. Validation of the new analytical solution

In order to validate the new analytical solution using shoreline data at Funatsu River delta, Eq. (3) is transformed into dimensional form as:

$$y = \frac{q0}{2\varepsilon DL}\left[\frac{x^2}{2} - Lx + \frac{L^2}{3} + \varepsilon t - \frac{2L^2}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}e^{-n^2\pi^2\frac{\varepsilon t}{L^2}}\cos\left(\frac{n\pi x}{L}\right)\right] \tag{11}$$

Figure 7 shows the evolution of the Funatsu River delta from 1982 to 2015 and the coordinate system used in the analysis. From several photos in Figure 7, it can be seen that there is no delta shape in 1982 at

Funatsu River mouth. Therefore, the year 1982 is used as the initial year from which the delta started to form owing to sediment supply from the river. Using the "delta-fitting" method presented by Duy et al. (2016) and replacing the equation provided by Larson et al. (1987) by Eq. (11), the parameters required for delta simulation at the Funatsu River delta are obtained in Table 1. Figure 8 shows the final result of the fitting process. In which, the values of q_0 and ε in Table 1 are changed to simulate different shoreline positions using Eq. (11). The simulated shoreline positions are compared with the measured shoreline in 1982. The root-mean-square error (RMSE) is calculated and the fitting process will stop when the smallest values of RMSE is obtained. In this case, RMSE=1.92 m.

After confirming the values of ε and q_0, the parameters in Table 1 are used to simulate the formation process of the Funatsu River mouth. In this step, the simulation is done using both (i) the new solution with the effect of the boundary (Eq. 11) and (ii) the solution provided by Larson et al. (1987). The shoreline evolutions near the boundary (x=-480 m) are compared between the simulated results and the measured data to see the effect of the boundary as shown in Figure 9. As can be seen from the figure, the new solution (blue line) shows better agreement with the measured shoreline positions near the boundary (x=-480 m). This result indicates that the new analytical solution (Eq. 11) is applicable for studying the formation processes of finite river delta shorelines.

4. Conclusions

Comparison between two analytical

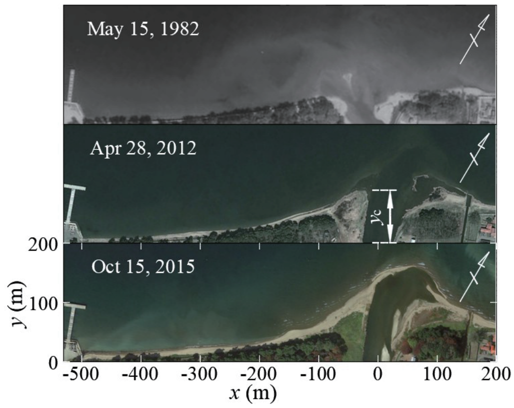

Figure 7. Evolution of Funatsu River delta and coordinate system used in the analysis.

Table 1. Parameters used for delta simulation at Funatsu River delta

Formation time (from 1982 to 2012)	t_0=30 years
Depth of closure (Fujita and Tanaka, 2004)	D_C=1.36 m
Beach length	L=490 m
Maximum shoreline position	y_C=84 m
Diffusion coefficient	ε=3 m²/day
Sediment supply from the river	q_0=1,350 m³/y

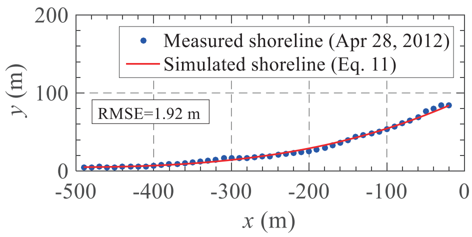

Figure 8. Fitting the measured shoreline and the theoretical shoreline to estimate q_0 and ε.

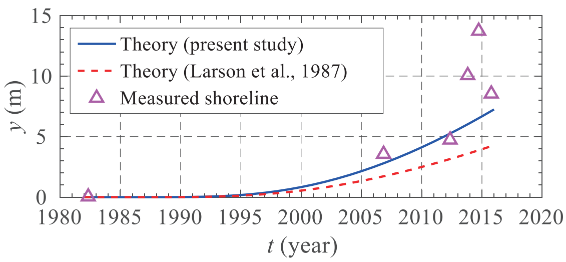

Figure 9. Shoreline evolutions at x=-480 m.

solutions provided by (i) Larson et al. (1987) and (ii) Tanaka et al. (2017) has been made to figure out the effect of boundaries to the formation processes of river delta shorelines. Using dimensionless forms, t^*=0.1 and t^*=0.3 are determined as the demarcations for the boundary to take effect at the river mouth and at the boundary, respectively. Using measured data, it can be said that the analytical solution provided by Tanaka et al. (2017) is suitable to examine the formation processes of finite river delta shorelines.

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